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AN AFFINE INVARIANT BIVARIATE VERSION OF
THE SIGN TEST

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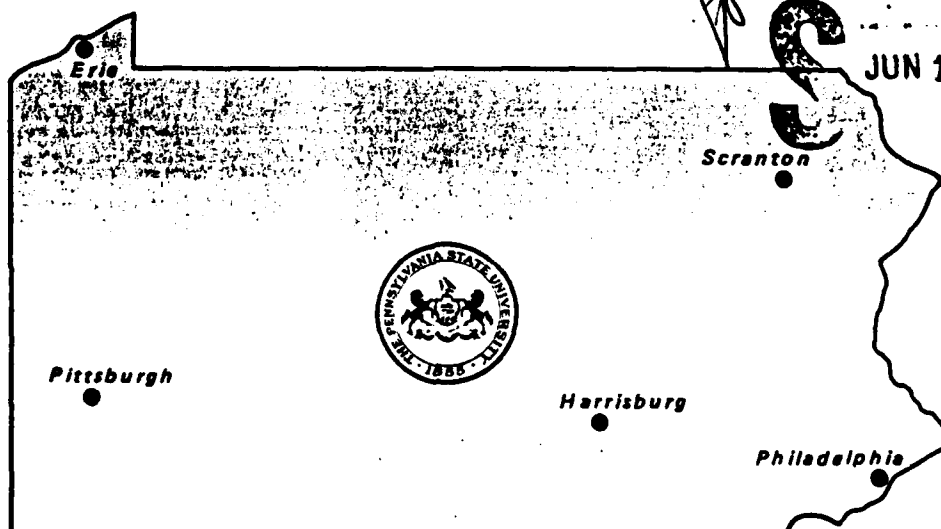87 6 16 012

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Summary

The generalized median of H. Oja yields a notion of bivariate quantile and in turn, an affine invariant bivariate analogue of the sign test. Its properties include a simple null covariance formula, facilitating a permutation or sign change test in the case of bivariate symmetry, normal efficiency coinciding with that of the Oja median, and bounded influence, hence strong robustness.

Key words: affine invariance, bivariate quantile, bivariate symmetry, model, generalized median, influence function, permutation test, normal efficiency, robustness, spatial median.

1. INTRODUCTION

The task of finding affine-invariant bivariate procedures which are analogues of univariate rank methods is not straightforward; see Barnett (1976) and the accompanying discussion for some of the difficulties. Recently, however Brown and Hettmansperger (1987) derived affine invariant bivariate analogues of both Wilcoxon rank-sum and signed rank tests, in bivariate two-sample and one-sample symmetric problems. This is done by combining the bivariate median of Oja (1983) with a linear-model approach of Jaeckel (1972). The Oja median minimizes an objective function which is the sum of areas of certain triangles and the gradients of this function yield a notion of bivariate quantile. The proposed tests are genuine Wilcoxon analogues through involving bivariate "quantile" rather than univariate rank. For tests in one and two-way layouts analogous to the Kruskal-Wallis and Friedman tests see Brown and Hettmansperger (1986).

In addition permutation tests are available through conventional arguments of sign change in the one-sample and permutation in the two-sample problem. Null covariance matrices have simple and easily computable forms, yielding convenient large sample normal or chi-squared approximations.

The gradient of the Oja objective function is just the bivariate quantile of the tested parameter vector, and when used as a test statistic, hereafter called the *Oja sign test* (OS test), should constitute a univariate sign test analogue. However, a sign change argument cannot be applied directly to the OS test to find its

conditional covariance matrix. Hence, at first sight, it appears that no convenient large sample test is available.

The present paper focuses on the one-sample case of bivariate symmetry, and does two things.

(i)   It is shown that the OS test can be re-written in a form which does show it to be a sign-test analogue. At the same time, its null covariance matrix reduces to a simple and easily computable form, and large-sample approximations are available.

(ii)   The test efficiency for normal sampling is derived, and shown to coincide with the normal estimation-efficiency of the Oja median (Oja and Niinimaa, 1985). It is also possible to calculate a bivariate version of the influence function, and the resulting form is bounded, as is the case for the univariate sign test, and shows the OS test to be of high robustness.

Sections 2 and 3,4 contain (i) and (ii) respectively.

2.   FORMULATION

Some material from Brown and Hettmansperger (1987) is now summarized briefly. Let $x_1, \ldots, x_n, \theta$ be 2x1 vectors with $\{x_i\}$ independent and with distribution symmetric about θ_0. The Oja objective function is

$$T(\theta) = \sum_{i<j} \sum A(x_i, x_j, \theta)$$

where $A(a,b,c)$ is the area of a triangle whose vertices are a,b,c. The Oja generalized median $\hat{\theta}$ is the choice of θ to minimize T. The quantile of θ is the vector whose components are derivatives of T with respect to components of θ; it is

$$Q(\theta) = \frac{1}{2} \sum_{i<j} \sum u(x_i, x_j; \theta) \tag{1}$$

where the "repulsion vector" $u(x_i, x_j; \theta)$ has magnitude $|x_i - x_j|$ and direction perpendicular to and away from the chord (x_i, x_j) towards θ. A bivariate sign test analogue uses $Q(\theta_0)$ to test $H_0: \theta = \theta_0$; in the univariate case the sign test statistic is exactly the centered quantile. The null hypothesis is rejected when $Q(\theta_0)$ is far from the zero vector, as measured by a quadratic form in $Q(\theta_0)$.

In what follows, take $\theta_0 = 0$ without loss of generality and let $u_{ij} = u(x_i, x_j; 0)$. $v_{ij} = u(x_i, -x_j; 0)$. The quantile Q as expressed in (1) is not an obvious sign-test analogue.

But write $u_{ij} = \frac{1}{2}\{(u_{ij} + v_{ij}) + (u_{ij} - v_{ij})\}$. Some simple geometry shows that

$$u_{ij} + v_{ij} = u(x_j, -x_j; -x_i)$$
$$u_{ij} - v_{ij} = u(x_i, -x_i; -x_j).$$

Substituting in (1) gives

$$Q(0) = \frac{1}{4} \sum_{i \neq j} \sum u(x_j, -x_j; -x_i) \tag{2}$$

$$= \frac{1}{4} \sum_i Q_i \tag{3}$$

where

$$Q_i = \sum_{\substack{j \\ (j \neq i)}} u(x_j, -x_j; -x_i).$$

An alternative expression for Q comes also from (2). Note that for fixed x_j. all $u(x_j, -x_j; -x_i)$ are perpendicular to $(-x_j, x_j)$ with magnitude $2|x_j|$. and direction determined by which side of the chord $(-x_j, x_j)$ the point $-x_i$ falls. Thus, let the extended chord $(-x_j, x_j)$

divide the plane into half-planes P_+, P_-: let a_j be the vector x_j rotated counter-clockwise through $\frac{1}{2}\pi$, so that $u(x_j, -x_j; -x_i) = \pm 2a_j$ for all i, and let P_+ be the half plane into which a_j points. Let r_j, s_j be the numbers of $\{-x_i, i \neq j\} \in P_+, P_-$ respectively. Then summing first over i in (2) yields

$$Q(0) = \frac{1}{2} \sum_j n_j a_j . \qquad (4)$$

where $n_j = r_j - s_j$.

The representation (4) may provide the best way to calculate Q, but the easiest derivation and computation of the covariance matrix of Q comes from (3). The analogue to the sign test is also best seen from (3), since each Q_i in some sense measures the position of x_i relative to the rest of the symmetrized sample.

Under the assumption of bivariate symmetry $\{x_i\}$ is a realization of $\{s_i x_i\}$ where $\{s_i\}$ are independent random variables each equalling ± 1 with probabilities $\frac{1}{2}, \frac{1}{2}$. Clearly Q_i depends on all $\{s_j\}$ only through s_i. Therefore, conditional on the collection $\{\pm x_j\}$, $Q_i = Q_i(x_i)$ and $Q_i(s_i x_i) = s_i Q_i(x_i)$, and (3) is a sum of independent random variables with vector coefficients. Thus $E(Q) = 0$ and the null covariance matrix of $Q(0)$ is

$$C = \frac{1}{16} \sum_i Q_i Q_i^T . \qquad (5)$$

A permutation or "sign-change" test against a general alternative may be carried out as follows. Let

$$Q_s = \frac{1}{4} \sum_i s_i Q_i .$$

Generate all 2^n possible values for $\{s_i\}$ and hence for Q_s: refer the observed value of $Q^T C^{-1} Q$ to the population of 2^n values of $Q_s^T C^{-1} Q_s$ and calculate a significiance level accordingly.

If n is large, this exact test can be modified to a Monte Carlo test where the $\{s_i\}$ are generated at random. Alternatively, a large sample approximation is available. Conditional on $\{Q_i\}$, each Q_s is the sum of independent random vectors and approximately normal for large n, with covariance matrix C. Thus the approximate null distribution to which $Q^T C^{-1} Q$ should be referred is χ_2^2.

3. EFFICIENCY

Tests of H_0: $\theta = \theta_0$ are based on $Q(\theta_0) = \frac{1}{2} \sum_{i<j} u(x_i, x_j; \theta_0)$. Again without loss of generality now take $\theta_0 = 0$. The null covariance matrix of Q(0) is $C = \frac{1}{16} \sum Q_i Q_i^T$; see (5). The actual test statistic is $Q^T(0)C^{-1}Q(0)$. Let $B = E(C)$; the U-statistic-like structure of C shows that $n^{-3}(C-B)$ converges almost surely to zero as $n \to \infty$, as long as $\{x_i\}$ are drawn from an integrable bivariate distribution. The asymptotic behaviours of $Q^T(0)C^{-1}Q(0)$ and $Q^T(0)B^{-1}Q(0)$ therefore coincide, and in assessing efficiency via a sequence of alternatives within $O(n^{-1/2})$ of the null, it is easy to show that the asymptotic distribution of $Q^T B^{-1} Q$ is noncentral χ_2^2 with noncentrality parameter

$$\beta^T D^T B^{-1} D\beta,$$

where $\theta = n^{-1/2}\beta$, $a = E_0\{Q(\theta)\}$ and D is the matrix of derivatives of a with respect to components of θ. Thus (see Bickel, 1965) the Pitman efficacy of the OS test appears to depend on the direction of the alternative, β. However, it will turn out that $D^T B^{-1} D$ is proportional to an orthogonal matrix (see Propositions 1 and 2), so the noncentrality parameter in fact does not depend on the direction of β. Taking β to be a unit vector, a large sample efficiency factor

for the test is just

$$e = \beta^T D^T B^{-1} D\beta, \tag{6}$$

which will equal the common eigenvalue of $D^T B^{-1} D$.

Now consider normal efficiency of the OS test relative to the least squares t-test. Both OS and t- test are affine invariant, so $\{x_i\}$ may be assumed independent $N(0, I_2)$, the bivariate circular normal distribution.

It is easy to verify that for least squares $D = I_2$, $B = n^{-1} I_2$, and hence that $e_{LS} = n$.

For the OS test, the calculation of e_{OS} is broken into two parts, first the calculation of a,D and second the calculation of B.

PROPOSITION 1. $\qquad\qquad D = \frac{n(n-1)}{2\pi} I_2.$

Proof. In this proof the direction of a line is taken to be an angle γ, or $\gamma+\pi$; that is, it is immaterial whether forwards or backwards orientation of the line is used.

First note that $x_i - x_j$ and $\frac{1}{2}(x_i + x_j)$ are independent, and hence that conditioning on the direction $\alpha \pm \frac{1}{2}\pi$ of $x_i - x_j$ does not influence $\frac{1}{2}(x_i + x_j)$. The repulsion vector $u(x_i, x_j; \theta)$ has direction α or $\alpha + \pi$. Since (x_i, x_j) and its negative are equi-probable, the contribution to $E\{u(x_i, x_j; \theta)\}$ from all (x_i, x_j) is zero apart from those (x_i, x_j) for which $u(x_i, x_j; \theta) = u(-x_i, -x_j; \theta)$. Projecting onto the direction α of the vector u, this condition means that the projection of θ cannot lie between those of the mid-points $\frac{1}{2}(x_i + x_j)$, $-\frac{1}{2}(x_i + x_j)$ of chords joining (x_i, x_j) and $(-x_i, -x_j)$. That is,

$$0 < \{\tfrac{1}{2}(x_i + x_j) - \theta\}^T \begin{bmatrix} \cos\alpha \\ \sin\alpha \end{bmatrix} \cdot \{-\tfrac{1}{2}(x_i + x_j) - \theta\}^T \begin{bmatrix} \cos\alpha \\ \sin\alpha \end{bmatrix}$$

$$= -|\tfrac{1}{2}(x_i + x_j)^T \begin{bmatrix} \cos\alpha \\ \sin\alpha \end{bmatrix}|^2 + |\theta^T \begin{bmatrix} \cos\alpha \\ \sin\alpha \end{bmatrix}|^2 ;$$

so the only x_i, x_j making non-zero contribution to $E\{u(x_i, x_j: \theta)\}$ satisfy

$$|\tfrac{1}{2}(x_i + x_j)^T \begin{bmatrix} \cos\alpha \\ \sin\alpha \end{bmatrix}| < |\theta^T \begin{bmatrix} \cos\alpha \\ \sin\alpha \end{bmatrix}| \qquad (7)$$

Recall that conditional on the direction α, i.e. on

$$(x_i - x_j)^T \begin{bmatrix} \cos\alpha \\ \sin\alpha \end{bmatrix} = 0,$$

$\tfrac{1}{2}(x_i + x_j) \simeq N(0, \tfrac{1}{2} I_2)$ so for small $|\theta|$, the probability of (7) is

$$2\pi^{-1/2} |\theta^T \begin{bmatrix} \cos\alpha \\ \sin\alpha \end{bmatrix}| + o(|\theta|) \qquad (8)$$

The vector $u(x_i, x_j: \theta)$ has magnitude $|x_i - x_j|$ and is in the direction $(\cos\alpha, \sin\alpha)^T$ with sign as yet unspecified. To get the sign, project both θ and the chord mid-point $\tfrac{1}{2}(x_i + x_j)$ on to the direction α, and note that u points away from the chord towards θ, by definition, so that

$$u(x_i, x_j: \theta) = |x_i - x_j| \begin{bmatrix} \cos\alpha \\ \sin\alpha \end{bmatrix} \operatorname{sgn}[\{\theta - \tfrac{1}{2}(x_i + x_j)\}^T \begin{bmatrix} \cos\alpha \\ \sin\alpha \end{bmatrix}].$$

However, for (x_i, x_j) obeying (7), the sign factor $= \operatorname{sgn}[(\cos\alpha, \sin\alpha)\theta]$. Since $E\{|x_i - x_j|\} = 2\pi^{-1/2}$, combining with (8) gives the corresponding contribution to $E\{Q(\theta)\}$ of

$$\frac{1}{2} \cdot \frac{4}{\pi} \quad \theta^T \begin{bmatrix} \cos\alpha \\ \sin\alpha \end{bmatrix} \cdot \begin{bmatrix} \cos\alpha \\ \sin\alpha \end{bmatrix} + o(|\theta|).$$

$$= \frac{2}{\pi} \begin{bmatrix} \cos^2\alpha & \cos\alpha \sin\alpha \\ \cos\alpha \sin\alpha & \sin^2\alpha \end{bmatrix} \theta + o(|\theta|).$$

Now α is a uniform angle, with $E(\cos^2\alpha) = \frac{1}{2} = E(\sin^2\alpha)$, $E(\cos\alpha \sin\alpha) = 0$, so finally

$$a(\theta) = \frac{2}{\pi} \frac{n(n-1)}{2} \frac{1}{2} \theta + o(|\theta|), \quad \text{and}$$

$$D = \frac{n(n-1)}{2\pi} I_2 \tag{9}$$

PROPOSITION 2.
$$B = \frac{n^3}{\pi^3} I_2 + o(n^3).$$

<u>Proof</u>. Referring to (5), a typical term of C is proportional to

$$u(x_j, -x_j; -x_i)u^T(x_k, -x_k; -x_i) + u(x_k, -x_k; -x_i)u^T(x_j, -x_j; -x_i).$$

Let $x_i^T = r_i\{\cos(\alpha_i + \frac{1}{2}\pi), \sin(\alpha_i + \frac{1}{2}\pi)\}$. By the independence of $\{r_i\}$, $\{\alpha_i\}$, the expectation of such a typical term, given x_j, x_k, is

$$4r_j r_k \begin{bmatrix} 2c_j c_k & (c_j s_k + c_k s_j) \\ (c_j s_k + c_k s_j) & 2s_j s_k \end{bmatrix} (1 - \frac{2}{\pi}|\alpha_j - \alpha_k|) \tag{10}$$

where $(c_i, s_i) = (\cos\alpha_i, \sin\alpha_i)$, and where in the last term, the factor $|\alpha_j - \alpha_k|$ is the shortest absolute rotation between α_j and α_k, and thus $\in [0, \pi]$. This factor arises from considering the possible positions of x_i in the four sectors defined by chords $(x_j, -x_j)$ and $(x_k, -x_k)$, and the corresponding signs of repulsion vectors u.

But $c_j c_k = \frac{1}{2}\{\cos(\alpha_j + \alpha_k) + \cos(\alpha_j - \alpha_k)\}$, $c_j s_k + c_k s_j = \sin(\alpha_j + \alpha_k)$, and $s_j s_k = \frac{1}{2}\{\cos(\alpha_j - \alpha_k) - \cos(\alpha_j + \alpha_k)\}$. Now $\alpha_i + \alpha_j$ and $\alpha_i - \alpha_j$ are independent and each is distributed as $U(-\pi, \pi)$, with cos and sin of $\alpha_i + \alpha_j$ having mean zero, so averaging over α_i, α_j reduces (10) to

$$4\ r_j r_k\ E\ [\cos(a_i - a_j) \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \{1 - \frac{2}{\pi}|\alpha_i - \alpha_j|\}].$$

$$= 4\ r_j r_k \cdot \frac{4}{\pi^2} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

Since $E(r_j) = E(r_k) = (2/\pi)^{1/2}$, the final result is:

$$B = \frac{n}{16} \left\{ \frac{(n-1)(n-2)}{2} \frac{32}{\pi^3} I_2 + O(1) \right\}$$

$$= \frac{n^3}{\pi^3} I_2 + o(n^3) \tag{11}.$$

Efficiency of the OS test compared to least squares

Applying the formula (6) with (9) and (11) gives

$$e_{OS} = \frac{n\pi}{4} + o(n).$$

and since $e_{LS} = n$, the required efficiency is

$$\text{efficiency (OS:LS)} = \frac{\pi}{4} = .785.$$

This efficiency is greater than $2/\pi = .637$, the normal efficiency of the univariate sign test, and the result agrees with the estimation efficiency of the Oja median for bivariate normal data; see Oja and Niinimaa (1985)

It is also possible to calculate the normal efficiency of the OS test relative to the componentwise sign test, a non-affine invariant test whose components are sign test statistics in the two co-ordinate directions. Consider bivariate normal data with covariance matrix

$$A^2 = \begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix}.$$

Then if X is $N(0, I_2)$, $Z = AX$ is $N(0, A^2)$. But by affine invariance, $e_{OS}(A) = e_{OS}(I_2)$.

Letting S be the vector of sign test statistics based on components of Z,

$$a = E_\theta(S) = n \begin{bmatrix} 1-2\Phi(\theta_1) \\ 1-2\Phi(\theta_2) \end{bmatrix}$$

$$= -n \, (2/\pi)^{1/2} \, \theta + o(|\theta|),$$

so that $D = -n \, (2/\pi)^{1/2} \, I_2$. Also, at $\theta = 0$

$$cov(S) = n \begin{bmatrix} 1 & 2\pi^{-1}\arcsin(\rho) \\ 2\pi^{-1}\arcsin(\rho) & 1 \end{bmatrix} = B.$$

The resulting expression for $D^T B^{-1} D$ has unequal eigenvalues $2\pi^{-1}n \, (1 \pm 2n^{-1}\arcsin \rho)^{-1}$ and the efficiency factor e_S for the componentwise sign test lies between these two values. The range of the resulting relative efficiencies is

$$\text{efficiency (OS:S)} = e_{OS}/e_S = \frac{\pi^2}{8} \, (1 \pm 2\pi^{-1} \arcsin (\rho))$$

with the actual efficiency depending on the direction of the alternative, as might be expected from the non-affine invariant nature of the S test. The median of these efficiencies is $\pi^2/8 = 1.234$, indicating that the OS test is generally more efficient than the componentwise sign test.

8. ROBUSTNESS

It is convenient to describe robustness of the OS test in terms
of a bivariate analogue of Hampel's (1974) influence function. The
latter, though usually defined as a von-Mises derivative of certain
functionals, can also be specified for tests, in the context of
asymptotic theory, in the following way, covering both bivariate and
univariate cases.

Let T be a normalized test statistic based on a large number n of
observations, whose null distribution is standard normal. Suppose
sampling is from a distribution contaminated at a fixed point x, that
is with probability $1-\epsilon$, sampling is from the hypothesized parent
distribution, but with probability ϵ, an observation is x. If
contamination is $O(n^{-1/2})$, i.e. $\epsilon = cn^{-1/2}$ for some $c > 0$, then
typically the asymptotic variance or covariance matrix of T is
unaffected, but the null mean $\rightarrow c\Omega$ as $n \rightarrow \infty$. The vector $\Omega = \Omega(x)$ is
the influence due to contamination at x; its presence imposes a bias
on the asymptotic null distribution and a distortion of test levels.

To evaluate Ω for the OS test, the null mean and covariance
matrix of Q(0) under $O(n^{-1/2})$ contamination are required; as before
assume the parent distribution to be $N(0, I_2)$.

It is easy to see that covariance is unaffected asymptotically,
and that as previously calculated

$$\mathrm{cov}\{Q(0)\} = B \simeq \frac{n^3}{\mathbf{T}^3} I_2 \qquad (12).$$

To evaluate $E\{Q(0)\}$, use the form (4); then

$$E\{Q(0)\} = \tfrac{1}{2}n(n-1) \, E(pa).$$

where if x_j is a typical observed vector, $|a| = |x_j|$ and a is perpendicular to x_j, pointing into the half plane P_+, and if p_+, p_- are probabilities of other observations in P_+, P_-, then $p = p_+ - p_-$. If x_j is from the contaminant x, then $p = 0$ but if x_j is from $N(0, I_2)$, then $p = \epsilon$ if $x \in P_+$, but $p = -\epsilon$ if $x \in P_-$. Averaging over x_j positions, using the independence of orthogonal components of $N(0, I_2)$ and the fact that expected absolute value of $N(0,1)$ is $(2/\pi)^{1/2}$ yields

$$E\{Q(0)\} = \frac{1}{2} n(n-1) \, \epsilon(1-\epsilon) \, (2/\pi)^{1/2} \, u_x,$$

where u_x is a unit vector in the direction of x.

Now let $\epsilon = cn^{-1/2}$ and calculate the mean of the normalized statistic $B^{-1/2}Q(0)$, i.e.

$$B^{-1/2} \frac{1}{2}n(n-1) \, cn^{-1/2}(1-cn^{-1/2}) \, (2/\pi)^{1/2} \, u_x$$

which from (12) approaches $2^{-1/2} \pi c \, u_x$ as $n \to \infty$. Thus for the OS test, the influence function is

$$\Omega(x) = 2^{-1/2} \pi u_x.$$

In depending only on the direction and not magnitude of the contaminant position x, this is analogous to influence for the univariate median. The factor $2^{-1/2}$ is attributable to the normal parent distribution. In having bounded influence, the OS test has high robustness.

Another bivariate sign test analogue, though not affine invariant, is the angle test, corresponding to the spatial median (see Brown, 1983). Similar but easier calculations show the standard normal influence function for angle tests to be

$$\Omega(x) = 2^{1/2} u_x.$$

which is also bounded, and median-analogous, with a smaller constant $2^{1/2}$. The larger constant $2^{-1/2}\pi$ for the OS test can be seen as a modest price to pay for the important property of affine invariance.

Other influence functions which are readily calculated are $\Omega(x) = x$ for Hotelling's test based on the bivariate sample mean, and therefore of unbounded influence as expected, and for the componentwise sign test, where the test vector has univariate sign tests as components, $\Omega(x) = 2^{1/2}u_x^*$, where u_x^* is a unit vector splitting the quadrant containing x. Thus the latter test, which is not affine invariant, also has bounded influence.

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

The generalized median of H. Oja yields a notion of bivariate quantile and in turn, an affine invariant bivariate analogue of the sign test. Its properties include a simple null covariance formula, facilitating a permutation or sign change test in the case of bivariate symmetry, normal efficiency coinciding with that of the Oja median, and bounded influence, hence strong robustness.

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